Of Copulas, Quantiles, Ranks and Spectra an L_1-approach to spectral analysis

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Abstract

In this paper we present an alternative nonparametric method for the spectral analysis of a strictly stationary time series $\{Y_t\}_{t\in\mathbb{Z}}$. We define a "new" spectrum as the Fourier transform of the differences between copulas of the pairs (Y_t, Y_{t-k}) and the independence copula. This object is called *copula spectral density kernel* and allows to separate marginal and serial aspects of a time series. We show that it is intrinsically related to the concept of quantile regression. Like in quantile regression, which provides more information about the conditional distribution than the classical location-scale model, the copula spectral density kernel is more informative than the spectral density obtained from the autocovariances. In particular the approach provides a complete description of the distributions of all pairs (Y_t, Y_{t-k}). Moreover, it inherits the robustness properties of classical quantile regression, because it does not require any distributional assumptions such as the existence of finite moments. In order to estimate the copula spectral density kernel we introduce rank-based Laplace periodograms which are calculated as bilinear forms of weighted L_1-projections of the ranks of the observed time series onto a harmonic regression model. We establish the asymptotic distribution of those periodograms, and the consistency of adequately smoothed versions. The finite-sample properties of the new methodology, and its potential for applications are briefly investigated by simulations and a short empirical example.

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