A Donsker Theorem for Lévy Measures

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Given n equidistant realisations of a Lévy process $(L_t, t \ge 0)$, a natural estimator \hat{N}_n for the distribution function N of the Lévy measure is constructed. Under a polynomial decay restriction on the characteristic function φ , a Donsker-type theorem is proved, that is, a functional central limit theorem for the process $\sqrt{n}(\hat{N}_n - N)$ in the space of bounded functions away from zero. The limit distribution is a generalised Brownian bridge process with bounded and continuous sample paths whose covariance structure depends on the Fourier-integral operator $\mathcal{F}^{-1}[1/\varphi(-\bullet)]$. The class of Lévy processes covered includes several relevant examples such as compound Poisson, Gamma and self-decomposable processes. Main ideas in the proof include establishing pseudo-locality of the Fourier-integral operator and recent techniques from smoothed empirical processes.