Institut für
Mathematische Stochastik

Georg–August–Universität

Institut für Mathematische Stochastik, Maschmühlenweg 8 - 10, 37073 Göttingen

2. November 2005

Einladung

Im Stochastik-Kolloquium spricht

Prof. Dr. Joachim Weickert

Universität des Saarlandes, Saarbrücken

über das Thema

What can we learn from two and four pixels ?

Der Vortrag findet statt am

Mittwoch, 30.11.2005 um 11:15 Uhr

im

Seminarraum der Stochastik, Maschmühlenweg 8 - 10

Es laden ein: die Dozenten des Instituts für Mathematische Stochastik

---

Abstract:
Signal and image denoising is a field where one is typically interested in removing noise without sacrificing important structures such as discontinuities. To this end, a large variety of nonlinear strategies has been proposed in the literature including wavelet shrinkage, nonlinear diffusion filtering and variational restoration. The goal of this talk is to analyse connections between these techniques that become particularly evident when considering signals of two pixels and images of four pixels.
We start with two-pixel signals, where we establish equivalence between soft Haar wavelet shrinkage, total variation (TV) diffusion, and TV regularization. By deriving identical analytical solutions of TV diffusion and TV regularisation of N-pixel signals, we prove the equivalence of both processes. The analytical solution in the two-pixel case can be used as a building block for a simple wavelet-inspired scheme for TV denoising. It is explicit, absolutely stable, conditionally consistent, and it does not require additional efforts to handle singularities gracefully.
Generalising these ideas to the 2-D setting leads to a novel, locally analytic scheme for TV diffusion and other diffusion processes with singular diffusivities. It is based on analytical solutions for images with four pixels and can be regarded as shift-invariant Haar wavelet shrinkage on a single scale with a new shrinkage rule. Compared to explicit schemes with regularised diffusivities, larger time step sizes can be used and sharper edges are obtained.
Finally we consider anisotropic, tensor-driven diffusion filters on four-pixel images. Fixing the diffusion tensor leads to dynamical systems with analytical solutions that can also be regarded as anisotropic Haar wavelet shrinkage. These processes can be assembled to a locally semianalytic scheme for anisotropic diffusion filtering that reveals a very high degree of rotation invariance and does not suffer from visible blurring artifacts. These results show that even simple Haar wavelets can be interesting alternatives to sophisticated wavelet concepts such as ridgelets and curvelets when anisotropic filtering is desired.

Joint work with Martin Welk, Gabriele Steidl and Thomas Brox.