Convergence rates for convex Tikhonov regularisation

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In classical Tikhonov regularisation for the stable solution of linear inverse and ill-posed problems, one uses quadratic penalty terms both for the residual term and for the regularisation term. One important question is then, to provide an estimate for the quality of the regularised solution in dependence of the noise level, the regularisation parameter, and the smoothness of the true (noise-free) solution of the problem. Here, the smoothness is measured in terms of source conditions defined by (fractional) powers of the operator that is approximately inverted. In many modern applications, however, quadratic regularisation is not appropriate, for instance in image processing or in situations where one is given additional statistical prior knowledge. In these situations, existence theory and questions like stability and convergence of Tikhonov regularisation are well established, but there are still several open questions concerning convergence rates, that is, precisely these quantitative estimates about the quality of the solution. One major problem is the definition of solution smoothness, because the quadratic approach by using rational powers of the operator is either inappropriate in the case of a non-quadratic setting on Hilbert spaces or, on general Banach spaces, not possible at all. In this talk, we will discuss, how solution smoothness and convergence rates in the convex Banach space setting can be tackled by using methods from convex analysis and duality theory.