Complexity of linear ill-posed problems in Hilbert space

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February 7, 2018

Information complexity of ill-posed problems may be seen as controversial. On the one hand side there were pessimistic results stating that the complexity is infinite, while on the other hand side the theory of ill-posed problems is well developed. In contrast to well-posed problems (continuous solution operators) the complexity analysis of ill-posed problems (discontinuous solution operators) is impossible without taking into account the impact of noise in the information. Commonly used models consider bounded deterministic noise and unbounded stochastic (Gaussian white) noise. It is common belief that white noise (a statistical ill-posed problem) makes ill-posed problems more complex than problems under bounded noise. In this study we shed light on a rigorous complexity analysis of ill-posed problems providing (tight) lower and upper bounds for both noise models. It will be shown that in contrast to the deterministic case statistical ill-posed problems have finite complexity at every prescribed error level. Moreover, the ill-posedness of the problem raises the issue of adaptation to unknown solution smoothness, and we provide results in this direction.
The presentation will first introduce the concept of 'Information-based complexity'. Then we will discuss how the paradigm of 'prescribed accuracy in reconstruction of the solution' changes the view on the ill-posedness, both for classical and statistical ill-posed problems.
This is joint work with Sergei V. Pereverzev, RICAM, Linz, published as Mathé, Peter; Pereverzev, Sergei V., "Complexity of linear ill-posed problems in Hilbert space.", J. Complexity 38 (2017), 50–67.