The Mittag–Leffler process and a scaling limit for the block counting process of the Bolthausen–Sznitman coalescent

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January 20, 2016

The Mittag–Leffler process $X = (X_t)_{t \geq 0}$ has the property that its marginal random variables X_t are Mittag–Leffler distributed with parameter e^{-t}, $t \in [0, \infty)$. The semigroup $(T_t)_{t \geq 0}$ of X satisfies $T_t f(x) = \mathbb{E}(f(x^{e^{-t}} X_t))$ for all $x \geq 0$ and all bounded measurable functions $f : [0, \infty) \to \mathbb{R}$. Some characteristics of the process X are derived, for example an explicit formula for the joint moments of its finite-dimensional distributions. The Mittag–Leffler process turns out to be Siegmund dual to Neveu's continuous-state branching process. The main result states that the block counting process of the Bolthausen–Sznitman n-coalescent, properly scaled, converges in the Skorohod topology to the Mittag–Leffler process X as the sample size n tends to infinity. A dual convergence result is obtained for the so called fixation line process of the Bolthausen–Sznitman coalescent. Generalizations to arbitrary exchangeable coalescents are briefly discussed.