

## Optimal Rank-Based Tests for Sphericity

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### Abstract

We propose a class of rank-based procedures for testing that the shape matrix of an elliptical distribution (with unspecified center of symmetry, scale, and radial density) is equal to some fixed value; this problem includes the problem of testing for sphericity as a particular case. The proposed tests are invariant under translations, monotone radial transformations, rotations, and reflections with respect to the estimated center of symmetry. They are valid without any moment assumption. For adequately chosen scores, they are locally asymptotically maximin (in the Le Cam sense) at given densities. The multivariate ranks used throughout are those of the distances—in the metric associated with the null value of the shape matrix—between the observations and the estimated center of the distribution. Local powers and asymptotic relative efficiencies with respect to the adjusted Mauchly test (i.e., a modified version of the Gaussian likelihood ratio procedure) proposed in Muirhead and Waternaux (1980) are derived.