Tracking Predictable Drifting Parameters of a Time Series

Dr. Paulo Serra
(Universität Göttingen)

November 27, 2013

Consider a time series $\{X_k,\, k \in \mathbb{N}_0\}$ where $X_0 \sim \mathbb{P}_0$ and subsequent observations X_k satisfy $X_k|X_{k-1} \sim \mathbb{P}_k(\cdot|X_{k-1})$, $k \in \mathbb{N}$, $X_{k-1} = (X_0, \ldots, X_{k-1})$. Suppose we are interested in certain (predictable) characteristics of the conditional distribution $\mathbb{P}_k(\cdot|x_{k-1})$: $\theta_k(x_{k-1}) = A_k(\mathbb{P}_k(\cdot|x_{k-1}))$ where the A_k are operators. Our time-varying parameter of interest $\theta_k(X_{k-1}) \in \Theta \subseteq \mathbb{R}^d$, $k \in \mathbb{N}$, is therefore a predictable process with respect to the natural filtration.

The parametric formulation is the simplest particular case of this setting: the observations are independent and $\theta \in \Theta$ is a constant vector. The simplest nonparametric formulation deals with independent observations and a time-varying parameter $\theta_k \in \Theta$, $k \in \mathbb{N}_0$. Markov chains with time varying transition laws are the next level of complexity.

Since data arrive in a successive manner, conventional methods based on samples of fixed size are not easy to use. Alternatively, stochastic recursive algorithms allow fast updating of parameter or state estimates at each instant as each new datum arrive. Our algorithm is based on the existence of a gain function – a (vector-valued) function of the previous estimate of $\theta_k(X_{k-1})$, a new observation X_k and prehistory X_{k-1} – which, roughly speaking, "pushes" the previous estimate towards the true parameter being tracked. A gain function, together with a step sequence and new observations from the model, can be used to adjust the current estimate, resulting in a recursive tracking algorithm.

To motivate our results we consider the problem of tracking conditional quantiles. We generalize the standard quantile regression model since we do not require observations to be independent. We also address the tracking of multidimensional analogs of the median, points of centrality or symmetry of multivariate distributions.

This is joint work with Eduard Belitser.