Probabilistic Approaches to Geometric Statistics

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Geometric statistics, the statistical analysis of manifold and Lie group valued data, has the Fréchet mean, a minimizer of the expected squared distance, as a fundamental construct. Similar least-squares criterions allow to generalize regression and principal component analysis beyond the Euclidean situation. An alternative to least-squares is to optimize the likelihood under families of parametric probability distributions on the nonlinear space. This probabilistic approach has maximum likelihood means as alternatives to the Fréchet mean, and it allows generalization of additional Euclidean statistical procedures that are defined via likelihoods. While parametric families of probability distributions are generally hard to construct in nonlinear spaces, transition densities of stochastic processes provide a geometrically natural way of defining likelihoods. In the talk, I will discuss common least-squares constructions in geometric statistics and their probabilistic counterparts, construction of geometrically natural probability distributions, and how curvature couples with the probabilistic models in distinctly non-Euclidean ways. Simulation of manifold and Lie group valued diffusion bridges here play an integral role in evaluation of likelihoods.