

## Lecture Series:

Financial data modelling across temporal scales:  
some remarks from the point of view of statistical  
information.

Prof. Marc Hoffmann

(École Nationale de la Statistique et de l'Administration Économique,  
Paris)

June 10, 2009, June 11, 2009, June 17, 2009 and June 18, 2009

In these four lectures, we will develop some statistical aspects of continuous time models which are addressed in finance.

**June 10, 2009, 12:00 – 13:00 in the Institut für Physik, Lecture Hall HS03, A0.105, Friedrich-Hund-Platz 1, 37077 Göttingen:**

In a first lecture, we will review some classical empirical facts that rule out simple diffusion models for describing the price dynamics of a financial asset. We will – partly – illustrate these evidences on real data examples. In particular, special focus will be made on scaling issues, in particular the appearance of microstructure noise at small temporal scales (below 30 minutes sampling, say). From a decision-theory point of view, we will discuss the non-equivalence between discrete GARCH(1,1) models and stochastic volatility diffusion models as obtained by Zhang and Brown in 2002, and therefore highlight the delicate issue of the deterioration of statistical information when only weak approximation of models are made.

**June 11, 2009, 14:15 – 15:15 in the Institut für Numerische und Angewandte Mathematik (NAM), Seminar Room (2nd floor), Lotzestr. 16-18, 37083 Göttingen:**

In a second lecture, we will specialize on financial modelling from coarse-to-fine scale, that is diffusion processes that are corrected – or partly corrected – at small scales by adding an extra-noise process that allows for reproducing microstructure noise. This approach is popular in financial econometrics and we will briefly review the accompanying – huge – literature. In this context, we will adopt a representation of the volatility in terms of building a non-parametric Gaussian limit experiments in the Le Cam sense, extending previous results of Gloter and Jacod (2000). This approach is non-constructive, and yields non-parametric asymptotic bounds that are complementary to recent results of Munk and Schmidt-Hieber (2008) in this direction.

**June 17, 2009, 11:15 – 12:15, in the Institut für Mathematische Stochastik (IMS), Seminar Room 5.101 (fifth floor), Goldschmidtstr. 7, 37077 Göttingen:**

The third lecture will be devoted to the reverse approach, when one models data from fine-to-coarse scales, starting with the order-book dynamics up to diffusion approximation. This aspect of statistical finance is mathematically not well established, and lay its roots in econophysics. We will attempt to draw a program of what could be done in this area from a statistical perspective, and highlight in particular the role of statistics of point processes.

**June 18, 2009, 14:15 – 15:15 in the Institut für Numerische und Angewandte Mathematik (NAM), Seminar Room (2nd floor), Lotzestr. 16-18, 37083 Göttingen:**

The fourth lecture will address the problem of estimating the (normalised) integrated volatility

$$\Lambda_t(\tau) := \tau^{-1} \int_t^{t+\tau} \sigma_s^2 ds, \quad t \geq 0$$

simultaneously for several temporal scales  $\tau > 0$  as we let  $t$  shift through time, and where  $\sigma_s$  is the volatility process of the underlying asset. This question can have some relevance in so-called variance trading or in statistical arbitrage issues. If the temporal scale  $\tau$  is large, the problem is essentially parametric, whereas at small time scales  $\tau$  it becomes genuinely nonparametric and depends on the underlying (generally unknown) smoothness of the volatility process. Besides, microstructure noise effects must be incorporated in a small  $\tau$  regime. We will develop a simple adaptive estimator based on wavelet thresholding for recovering  $\Lambda_t(\tau)$  at all scales  $\tau$ , optimal in a certain sense.