Goodness of Fit via Pointwise Likelihood Ratios: Some Results of Berk and Jones Revisited

Talk to be given at Göttingen
Institute for Mathematical Stochastics
March 9, 2005

by

Jon A. Wellner
Department of Statistics, University of Washington, Seattle WA

Abstract: Consider the classical "goodness-of-fit" testing problem: based on a sample of random variables $X_1, \ldots, X_n$ from a distribution function F on $\mathbb{R}$, we want to test the hull hypothesis $H: \; F(x) = F_0(x)$ for all $x \in \mathbb{R}$ versus $K: \; F(x) \neq F_0(x)$ for some $x \in \mathbb{R}$ where F_0 is a specified (continuous) distribution function. Berk and Jones (1979) proposed a test statistic based on the supremum of the pointwise log-likelihood ratio statistics

$$\log \lambda_n(x) = n\mathbb{F}_n(x) \log \left(\frac{\mathbb{F}_n(x)}{F_0(x)} \right) + n(1 - \mathbb{F}_n(x)) \log \left(\frac{1 - \mathbb{F}_n(x)}{1 - F_0(x)} \right)$$

for testing $H_x: \; F(x) = F_0(x)$ versus $K_x: \; F(x) \neq F_0(x)$. Thus the Berk-Jones statistic R_n is given by

$$R_n = \sup_{x \in \mathbb{R}} \log \lambda_n(x) \, .$$

Berk and Jones (1979) established the behavior of their test statistic under the null hypothesis H and show that it has some remarkable efficiency properties under fixed alternatives. Owen (1995) showed that the test of Berk and Jones can be inverted to obtain confidence bands for an unknown distribution function F.

In this talk I will review the results of Berk and Jones and discuss some new complementary results.