

# Adaptive Nonparametric Confidence Bands

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June 23 - 25, 2009

## **Part I: A very short introduction to nonparametric function estimation**

I shall review some of the main ideas of nonparametric function estimation in statistics, with a particular view on density estimation, but covering regression, simple statistical inverse problems, and distribution function estimation. I shall then define the notion of a nonparametric confidence band and discuss why this leads to an interest in the study of the distribution of the sup-norm risk of a nonparametric estimator. I shall also lay out some of the main classical ideas for nonparametric confidence bands going back to Smirnov (1950) and Bickel and Rosenblatt (1973), and some more recent ones based on Rademacher processes.

## **Part II: Adaptive Estimation in Supremum-Norm Loss**

I shall describe some of the main ideas behind 'adaptive' nonparametric function estimation, which gives a minimax approach to the choice of the tuning parameters that appear in classical nonparametric estimation procedures. The discussion will include Lepski's method, wavelet thresholding and model selection techniques. I shall then discuss recent results that establish adaptive risk bounds in sup-norm loss (so are relevant for confidence bands), but I shall also present a negative result due to Mark Low that shows that adaptive confidence bands do NOT exist in a certain sense, and which shows that there is a 'hiatus' between adaptive estimation and inference.

## **Part III: Adaptive Inference and Confidence Bands**

I shall try to tackle Low's problem by formulating a new framework for adaptive inference (in contrast to adaptive estimation). This framework is based on relating impossibility results for adaptive confidence sets to certain nonparametric testing problems studied by Ingster. This rather subtle connection allows to formulate the notion of 'a statistically optimal adaptive confidence band', so that the problem of when adaptive inference is possible can be completely characterized. I shall then give some further arguments for why such a notion of 'adaptive inference' is sensible for statisticians, based on a Bayesian and topological interpretation of the 'pathological' situations where adaptive inference is not possible.

The first talk (Wed. June 23, 11:15 – 12:15) and third talk (Fri. June 25, 14:15 – 15:15) are held in the Seminar Room of the IMS. The second (Thu. June 24, 14:15 – 15:15) is held in the Institut für Numerische und Angewandte Mathematik (NAM), Seminar Room (2nd floor), Lotzestr. 16-18, 37083 Göttingen, <http://www.num.math.uni-goettingen.de>