NON-PARAMETRIC FUNCTION ESTIMATION DEFINED ON A CONVEX DOMAIN

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Abstract

We consider the problem of estimating a bivariate function $f(x,y)$ supported on a convex set of R^2, given a discrete and noisy data recorded on a regular square grid. In particular circular and elliptical domains are taken into account or more generally any convex domains with both smooth and non-smooth boundaries. Such a setup is common in pattern recognition, medical imaging, biometric systems, and optometry. An estimate of $f(x,y)$ based on a class of orthogonal and complete functions over the domain is proposed. This basis has a distinctive property of being invariant to rotation of axes about the origin of coordinates yielding therefore a rotationally invariant estimate. For radial image functions the orthogonal set has a particularly simple form being related to the classical Legendre polynomials. We give the statistical accuracy analysis of the proposed estimate of $f(x,y)$ in the sense of the L_2 metric. A rate of converegence for the L_2 error is established assuming that $f(x,y)$ belongs to a class of bounded variation functions. It is found that there is an inherent limitation in the precision of the estimate due to the geometric nature of the convex domain. This is explained by relating the accuracy issue to the celebrated problem in the analytic number theory called the lattice points of a convex domain. In fact we show that this geometric error has a dominant influence on the overall performance if the boundary of the support set is not sufficiently smooth. This is a distinctive feature of the image reconstruction problem on a convex domain based on gridded data.