Some Frequentist Results About Posterior Distributions on Infinite-Dimensional Parameter Spaces

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In semiparametric and nonparametric statistics the unknown parameter is a function, or more generally an element of some infinite-dimensional Banach space. A Bayesian method starts, as usual, by the specification of a prior distribution on the parameter. In the case that the parameter is a function this is equivalent to modelling this function as a sample path of a stochastic process. Next the statistical model is viewed as a specification of the distribution of the data given the parameter, and Bayes' rule gives the conditional distribution of the parameter given the data, called posterior distribution.

We study the posterior distribution for various priors under the (nonBayesian) assumption that the data is generated according to some fixed true distribution. For instance, we are interested in whether, and if so how fast, a sequence of posterior distributions contracts to the true distribution if the amount of data increases. We review general results and various examples.

One of the attractions of Bayesian procedures is that it gives an elegant and principled framework for regularization and adaptation. As one example we shall discuss using a Gaussian process as a prior distribution. If we start with an infinitely smooth, stationary process, and scale this by an inverse Gamma variable, then we obtain a prior distribution on functions such that the posterior distribution adapts automatically to an unknown smoothness level in the true parameter. As another example we discuss adaptation to sparsity.

The first talk (Wed. Jan 21, 2009) is intended to be a not-too-technical overview, whereas the second talk (Thu. Jan 22, 2009 in the Graduate School) will also give insight in some of the methods of proof.

Second talk: Jan. 22, 2009, 14:15 – 15:15 in the Institut für Numerische und Angewandte Mathematik (NAM), Seminar Room (2nd floor), Lotzestr. 16-18, 37083 Göttingen, http://www.num.math.uni-goettingen.de