Degree-penalized contact processes
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The contact process is a model for the spread of an infection in a graph. Vertices can be either healthy or infected; infected vertices recover with rate 1 and send the infection to each neighbor with rate lambda. A key question of interest is: if we start the process with a single infected vertex, can the infection survive forever with positive probability? This typically depends on the graph and on the value of lambda; for instance, on integer lattices, there is a critical value of lambda at which the survival probability changes from zero to strictly positive. However, on graphs that include vertices of high degree, such as Galton-Watson trees with heavy-tailed offspring distributions, it has been observed that the infection survives with positive probability for all values of lambda, no matter how small. This is because high-degree vertices sustain the infection for a long time and send the infection to each other. In this work, we investigate this survival-for-all-lambda phenomenon for a modification of the contact process, which we introduce and call the penalized contact process. In this new process, vertex u transmits the infection to neighboring vertex v with rate $\lambda / \max(degree(u), degree(v))^{\mu}$, where $\mu > 0$ is an additional parameter (called the penalization exponent). This is inspired by considerations from social network science: people with many contacts do not have the time to infect their neighbors at the same rate as people with fewer contacts. We show that the introduction of this penalty factor introduces a rich range of behavior for the phase diagram of the contact process on Galton-Watson trees. We also show corresponding results for the penalized contact process on finite graphs obtained from the configuration model, which locally converge to Galton-Watson trees. Join work with Julia Komjathy and Zsolt Bartha.