Compressed sensing bounds via improved estimates for Rademacher chaos

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The theory of compressed sensing considers the following problem: Let $A \in \mathbb{C}^{m\times n}$ and let $x \in \mathbb{C}^n$ be s-sparse, i.e., $x_i = 0$ for all but s indices i. One seeks to recover x uniquely and efficiently from linear measurements $y = Ax$, although $m \ll n$. A sufficient condition to ensure that this is possible is the Restricted Isometry Property (RIP). A is said to have the RIP, if its restriction to any small subset of the columns acts almost like an isometry. We study matrices A with respect to the RIP which represent the convolution with a random vector followed by a restriction to an arbitrary fixed set of entries. We focus on the scenario that ϵ is a Rademacher vector, i.e., a vector whose entries are independent random signs.

We reduce this question to estimating random variables of the form

$$D_{\mathcal{A}} := \sup_{A\in\mathcal{A}} \left| \|A\epsilon\|^2 - \mathbb{E}\|A\epsilon\|^2 \right|,$$

where $\mathcal{A}$ is a set of matrices. Random variables of this type are closely related to suprema of chaos processes. Using generic chaining techniques, we derive a bound for $\mathbb{E}D_{\mathcal{A}}$ in terms of the Talagrand γ_2-functional. As a consequence, we obtain that the matrices under consideration have the RIP with high probability if the embedding dimension satisfies $m \geq Cs\log(n)^4$. This bound exhibits optimal dependence on s, while previous works had only obtained a suboptimal scaling of $s^{3/2}$.

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