Distance Methods for Point Processes

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I present an overview of my research on metrics between spatial point patterns and metrics between point process distributions.

For this talk we restrict ourselves to a compact state space $\mathcal{W} \subset \mathbb{R}^d$. A point process Ξ on $\mathcal{W}$ is a random element in the space of finite counting measures on $\mathcal{W}$. If we rule out the possibility to have mass > 1 ("multiple points") at a single location, we can interpret the realizations of Ξ as finite subsets of $\mathcal{W}$, which we call point patterns. A Poisson process H is a particularly pleasant point process, in which the point counts $H(A)$ and $H(B)$ of any pair of disjoint measurable sets $A, B \subset \mathcal{W}$ are independent.

In a first part of the talk an introduction to the statistics of point processes is given, including various key characteristics like moment measures and the conditional intensity. Then several metrics between point process distributions are defined and compared with one another. In this context we also study metrics between point patterns and discuss their applications in spatial statistics.

The remainder of the talk is devoted to distance estimates for Poisson process approximation. We first formulate a theorem that gives an upper bound in terms of the key characteristics introduced before for the distance between a rather general point process Ξ and the Poisson process H with the same expectation measure. The proof of this bound is based on Stein's method, a very versatile technique for obtaining distance estimates for probability distributions. We then consider a more concrete situation, where Ξ is obtained by dependently thinning points from another point process, and give an explicit rate of convergence to a Poisson process for a point process that is more and more contracted and covered by random balls.