FAST FOURIER TRANSFORM FOR NONEQUISPACED DATA WITH APPLICATIONS IN MRI AND NMR.

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In this talk we give an introduction in the fast Fourier transform for nonequispaced data (NFFT), show the commons with the gridding reconstruction in MRI and extend this approach to field inhomogeneity correction.

In the second part we discuss the following approximation problem with applications in NMR: Let h be a short linear combination of nonincreasing exponentials with complex exponents. Determine all exponents, all coefficients, and the number of summands from finitely many equispaced sampled data of h.

FAST FOURIER TRANSFORM AT NONEQUISPACED KNOTS AND APPLICATIONS

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We use the recently developed fast Fourier transform at nonequispaced knots (NFFT) in a variety of applications. The NFFT realized the fast computation of the sums

$$f(w_j) = \sum_{k=-n/2}^{n/2-1} f_k \, \mathrm{e}^{-2\pi \mathrm{i} k w_j} \qquad (j = -M/2, \ldots, M/2-1)$$

where $w_j \in [-1/2, 1/2)$. We describe the basic idea of the algorithm and discuss two important applications in detail.

Interpolation of scattered data. We discuss fast and reliable algorithm for the optimal interpolation of scattered data on the torus $\mathbb{T}^d$ by multivariate trigonometric polynomials as well as the approximation problem. The algorithm is based on a variant of the conjugate gradient method in combination with the NFFT. We present a worst case analysis as well as results based on probabilistic arguments. The main result is that under mild assumptions the total complexity for solving the interpolation or approximation problem at M arbitrary nodes is of order $\mathcal{O}(N^d \log N + M)$.

Fast summation. The fast computation of special structured discrete sums

$$f(y_j) := \sum_{k=1}^{N} \alpha_k K(\|y_j - x_k\|) \quad (j = 1, \ldots, M)$$

or from the linear algebra point of view of products of vectors with special structured dense matrices is a frequently appearing task. We develop a new algorithm for the fast computation of discrete sums based on NFFTs. Our algorithm, in particular our regularisation procedure, is simply structured and can easily be adapted to different kernels K, e.g.

$$\frac{1}{x}, \, \frac{1}{x^2}, \, x^2 \log x, \, \log x, \, (x^2 + c^2)^{\pm 1/2}.$$

We prove error estimates to obtain clues about the choice of the involved parameters. Furthermore we generalise this method to the sphere $\mathbb{S}^2$ and the rotation group $SO(3)$.