Elements of limit theory for von Mises statistics of measure preserving transformations

Prof. Mikhail Gordin
(V. A. Steklov Institut für Mathematik der Russischen Akademie der Wissenschaften, Abteilung Sankt Petersburg)

October 22, 2014

We will describe an approach, unifying and simplifying the study of limiting behavior of von Mises statistics for a wide class of stationary processes.
Every such a statistic is defined by a kernel, which is, informally speaking, a function of several variables. Giving a precise definition of classes of suitable kernels is a part of the problem we are dealing with. We define such classes as projective tensor products of some spaces L_p. Kernels from these classes can be identified with functions of several variables which are shown to admit a correct restriction to some subsets of measure zero of the cartesian powers of the main probability space (for example, to the principal diagonal of such a power). With these definitions in hand, the Strong Law of Large Numbers can be established for appropriate classes of von Mises statistics.
To prove some forms of the Central Limit Theorem (CLT) for von Mises statistics we assume that a filtration compatible with the time evolution is specified on the main probability space. Then we apply martingale approximation, using the so-called martingale/coboundary decompostion in its recent multiparameter version, to prove the CLT for non-degenerate von Mises statistics and for totally degenerate (or canonical) von Mises statistics of degree two.
Possible further developments and interrelation with some previously known results (old and recent) on von Mises statistics for dependent variables will be briefly discussed.
The talk is mainly based on a joint paper with Manfred Denker.