# The Mittag–Leffler process and a scaling limit for the block counting process of the Bolthausen–Sznitman coalescent

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The Mittag–Leffler process $X = (X_t)_{t\geq 0}$ has the property that its marginal random variables $X_t$ are Mittag–Leffler distributed with parameter $e^{-t}$, $t \in [0,\infty)$. The semigroup $(T_t)_{t\geq 0}$ of $X$ satisfies $T_t f(x) = \mathbb{E}(f(x^{e^{-t}} X_t))$ for all $x \geq 0$ and all bounded measurable functions $f : [0,\infty) \to \mathbb{R}$. Some characteristics of the process $X$ are derived, for example an explicit formula for the joint moments of its finite-dimensional distributions. The Mittag–Leffler process turns out to be Siegmund dual to Neveu's continuous-state branching process. The main result states that the block counting process of the Bolthausen–Sznitman $n$-coalescent, properly scaled, converges in the Skorohod topology to the Mittag–Leffler process $X$ as the sample size $n$ tends to infinity. A dual convergence result is obtained for the so called fixation line process of the Bolthausen–Sznitman coalescent. Generalizations to arbitrary exchangeable coalescents are briefly discussed.