On the Smallest Dimension and the Largest Margin of Linear Arrangements for Matrices

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Abstract

A matrix $M \in \{-1,1\}^{m \times n}$ can be represented by vectors $u_1, \ldots, u_m, v_1, \ldots, v_n \in \mathbb{R}^k$ such that the sign of the inner product of u_i and v_j coincides with $M_{i,j}$. We may view $u_1, \ldots, u_m$ as a linear arrangement of hyperplanes and $v_1, \ldots, v_n$ as points located in the cells of this arrangement. The smallest k such that a k-dimensional arrangement for M exists is denoted as $d(M)$. The distance between the union of points $v_1, \ldots, v_n$ and the union of hyperplanes induced by $u_1, \ldots, u_m$ is called the (minimum) margin of the arrangement. The largest possible minimum margin of an arrangement for M is denoted as $\mu(M)$. The problem of finding a low-dimensional arrangement or an arrangement with large margin has important applications in learning and complexity theory. In the talk, we discuss some recent bounds on $d(M)$ and $\mu(M)$. The main algebraic tools are the spectral norm and the singular value decomposition. We also mention some implications of the results in learning and complexity theory.